Multi Objective–Multi Item Inventory Model Using Pentagonal Fuzzy Number

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Abstract—*In this paper, multi objective - multi item inventory model is developed in a fuzzy environment. The objectives are to maximize the average total profit and to minimize the average total cost for store and warehouse. The pentagonal fuzzy number is defined and its properties are given. All the cost parameters are fuzzy in nature and are represented by Pentagonal fuzzy number. The objective functions are defuzzified using nearest interval approximation method. The optimal economic order quantity and optimal display inventory level are derived by using fuzzy geometric programming technique. This model is illustrated with a numerical example.*

Keywords*: Multi objective, Multi item, Pentagonal fuzzy number, Geometric Programming technique.*
AMS Subject Classification (2010): 90B05

1. INTRODUCTION

Multi – item classical inventory models under various types of constraints such as capital investment, available storage area, number of orders and available set – up time are presented in well – known books [3], [7] and [13]; etc..

While modeling an inventory problem, generally three types of demand are considered. They are (1) Constant demand (2) Time – dependent demand and (3) Stock – dependent demand. In the stock dependent demand, specially displayed inventory level demand has an effect on sales for many retail products.

The inventory control problem for style goods is further complicated with the fact that inventory and sales are not independent of one another is explained in [16]. An increase in inventory may bring about increased sales of some items. The sale at the retail level is proportional to the amount of displayed inventory is given in [13]. The most of the retailers displayed some products on shelf following the product variety, customer's choice of brand quality, and physical size of the product to influence the customer's attention.

A model to identify those products, which should be included in a firm's product line in which the demand rate is a polynomial function of price, advertising and distribution is developed in [15]. A shelf – space allocation model in which demand rate is a function of shelf – space allocated to the product is published in [4].

But all these inventory problems are solved with the assumption that the co-efficient or cost parameters are specified in a precise way. In real life, there are many diverse situations due to uncertainty. Here inventory costs are imprecise, that is fuzzy in nature.

Early works in using fuzzy concept in decision making were done in [18]. Fuzzy goals, costs and constraints were introduced in [2]. Later, the fuzzy linear programming model was formulated and an approach for solving linear programming model with fuzzy numbers has been presented in [19].

Geometric programming method is a relatively new technique to solve a non-linear programming problem. The idea on GP method is first developed in [5]. A multi-item inventory model with several constraints using posynomial GP method analyzed in [17]. Later, the Geometric Programming techniques were discussed in [1], [8], [9], [11] & [12]. In [11], a displayed inventory model with triangular fuzzy number is presented. Demand rate is dependent on the displayed inventory level and there is a limitation on total display space.

Recently, an inventory problem of deteriorating items with three objectives- maximization of profit and minimization of total average cost and wastage cost in fuzzy environment is presented in [20].

Lately, power scarcity is affecting the small scale industries such as Bakery, Restaurants, Packaged food product companies, Retail showrooms etc. To solve this problem, generators are being installed, it incurs a cost. This paper introduces/refers the cost as 'Alternative power supply cost'. Power generator has been used in both backroom storage area and display area. Also the pentagonal fuzzy number is defined and its properties are given.

All the cost parameters involved in the objective functions are fuzzy in nature and are represented by Pentagonal fuzzy number. The objective functions are defuzzified using nearest interval approximation method. The optimal economic order quantity and optimal display inventory level are derived by using fuzzy geometric programming technique. This model is illustrated with a numerical example.

2. ASSUMPTIONS AND NOTATIONS:

A multi objective multi item displayed inventory model with generator cost is formulated under the following assumptions and notations.

Assumptions

1. The unit cost of the item is independent of order quantity Q.
2. The display cost does not depend on the length of cycle time T.
3. The outstanding order was never more than one.
4. Lead time is zero.
5. Shortages are not allowed.
6. Demand rate depends on display inventory for i^{th} item

$D_i = d_i \; S_i^{d_i'} \quad (d_i > 0, \;\; 0 < d_i' < 1)$

Here d_i and d_i' (i=1, 2,….., n) are scale

and shape parameters of the demand

function

7. Full-shelf merchandising policy has been adopted, where the display area is always kept fully stocked, so the inventory is replenished as soon as the backroom inventory reaches zero. The displayed inventory will always be at its maximum. The inventory level decreases at a constant rate.
8. Alternative power supply (power generator) cost is allowed.

Notations

The following are for i[th] item (i = 1,2,3.......n)

n – number of items

S_i – number of display quantity (decision variable) Q_i – number of order quantity (decision variable)

D_i – demand rate

$\tilde{P}_i$ - fuzzy production rate

$\tilde{p}_i$ - fuzzy selling price per unit per cycle

$\tilde{C}_i$ - fuzzy purchasing cost per unit per cycle

$\tilde{C}_{2i}$ - fuzzy display shelf cost per unit per unit time

$\tilde{C}_{3i}$ - fuzzy set up cost per cycle

$\tilde{C}_{w1i}$ - fuzzy holding cost per unit per unit time for warehouse

$\tilde{g}_{wi}$ - fuzzy alternative power supply cost per unit per unit time for warehouse

$\tilde{t}_{wi}$ - fuzzy transportation cost for warehouse

$\tilde{C}_{s1i}$ - fuzzy holding cost per unit per unit time for store

$\tilde{g}_{si}$ - fuzzy alternative power supply cost per unit per unit time for store

$\tilde{C}_{1i}$ - total fuzzy holding cost per unit per unit time $= C_{w1i} + C_{s1i}$

$\tilde{g}_i$ - total fuzzy alternative power supply cost per unit per unit time $= g_{wi} + g_{si}$

$\tilde{P}F$ – fuzzy average profit function

$\tilde{T}C$ – fuzzy average total cost for store

$\tilde{W}C$ – fuzzy average total cost for warehouse

$T_i =$ cycle time $= \dfrac{Q_i}{d_i S_i^{d_i^1}}$

θ_i - instantaneous inventory level of the entire system including both the back room storage and the display inventory.

3. MATHEMATICAL MODEL IN CRISP ENVIRONMENT

Our proposed model is to maximize the average profit and to minimize the average total cost for store and warehouse.

$$Max\ PF(S,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} d_i S_i^{d_i'} (p_i - C_i) - \dfrac{C_{3i}}{Q_i} d_i S_i^{d_i'} \\ - \dfrac{(g_i + C_{1i})\, Q_i}{2} + \dfrac{d_i S_i^{d_i'} (C_{1i} + g_i) Q_i}{2P_i} \\ -(g_i + C_{1i} + C_{2i})\, S_i + \dfrac{d_i S_i^{d_i'+1} (C_{1i} + g_i)}{P_i} \end{array} \right] \quad (1)$$

$$Min \quad Tc\,(S,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} \dfrac{C_{3i}}{Q_i} d_i\, S_i^{d_i'} + \dfrac{(g_{si} + C_{s1i})\, Q_i}{2} \\ - \dfrac{d_i\, S_i^{d_i'} (C_{s1i} + g_{si}) Q_i}{2P_i} \\ + (g_{si} + C_{s1i} + C_{2i})\, S_i \\ - \dfrac{d_i S_i^{d_i'+1} (C_{s1i} + g_{si})}{P_i} \end{array} \right] \quad (2)$$

$$Min \quad Wc(S,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} \dfrac{(g_{wi} + C_{w1i})\, Q_i}{2} - \\ \dfrac{d_i\, S_i^{'\,d_i}\, (C_{w1i} + g_{wi})\, Q_i}{2P_i} + \\ \dfrac{t_{wi}\, d_i\, S_i^{'\,d_i}}{Q_i} \end{array} \right] \qquad (3)$$

The multi-objective programming problems have been considered as a single objective problem and it is solved by geometric programming technique. That is only one objective function is taken at a time and ignoring the rest objective functions. Let X^i be the optimal solution when only i^{th} objective function is considered as objective function. The result is shown in matrix [P] as follows

$$\begin{array}{cccc} & PF & TC & WC \\ X^1 & PF^1 & TC^1 & WC^1 \\ X^2 & PF^2 & TC^2 & WC^2 \\ X^3 & PF^3 & TC^3 & WC^3 \end{array}$$

Here, The Diagonal Elements represent the optimal values of the corresponding objectives.

4. PENTAGONAL FUZZY NUMBER AND ITS NEAREST INTERVAL APPROXIMATION:

Definition 4.1: A pentagonal fuzzy number $\widetilde{A}$ described as a fuzzy subset on the real line R whose membership function $\mu_{\widetilde{A}}(x)$ is defined as follows

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0 & , \quad a > x \\ w_A \dfrac{x-a}{b-a} & , \quad a \le x \le b \\ w_A + (1-w_A)\dfrac{x-b}{c-b} & , \quad b \le x \le c \\ w_A + (1-w_A)\dfrac{d-x}{d-c} & , \quad c \le x \le d \\ w_A \dfrac{e-x}{e-d} & , \quad d \le x \le e \\ 0 & , \quad e < x \end{cases}$$

where $0.6 \le w_A < 1$ and a, b, c, d and e are real numbers.

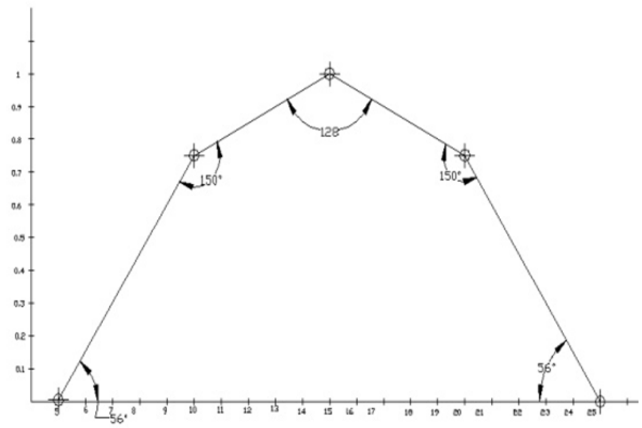

Fig. 1: Graphical representation of Pentagonal Fuzzy number for w_A = 0.75

This type of fuzzy number be denoted as

$\widetilde{A}$ = (a, b, c, d, e; w_A) PFN

$\mu_{\widetilde{A}}$ satisfies the following conditions:

1. $\mu_{\widetilde{A}}$ is a continuous mapping from R to the closed interval [0, 1].
2. $\mu_{\widetilde{A}}$ is a convex function.
3. $\mu_{\widetilde{A}} = 0, -\infty < x \le a.$
4. $\mu_{\widetilde{A}}$(x) = L(x) is strictly increasing on (a, c).
5. $\mu_{\widetilde{A}}$(x) = 1, x=c.
6. $\mu_{\widetilde{A}}$(x) = R(x) is strictly decreasing on (c, e).
7. $\mu_{\widetilde{A}}$(x) = 0, $e \le x < \infty$.

Remarks:

1. If w_A < 0.6 then $\widetilde{A}$ becomes a triangular fuzzy number.

2. If w_A =1 then $\widetilde{A}$ becomes a trapezoidal fuzzy number.

Nearest Interval Approximation: Suppose $\widetilde{A}$ and $\widetilde{B}$ are two fuzzy numbers with α -cuts are $[A_L(\alpha), A_R(\alpha)]$ and $[B_L(\alpha), B_R(\alpha)]$, respectively. Then the distance between $\widetilde{A}$ and $\widetilde{B}$ is

$$d(\widetilde{A},\widetilde{B}) = \sqrt{\int_0^1 (A_L(\alpha) - B_L(\alpha))^2\, d\alpha + \int_0^1 (A_R(\alpha) - B_R(\alpha))^2\, d\alpha}$$

Given $\widetilde{A}$ is a pentagonal fuzzy number. To find a closed interval $C_d(\widetilde{A})$, which is the nearest to $\widetilde{A}$ with respect to metric d. Since each interval is also a fuzzy number with constant α -cut for all α ∈ [0,1]. Hence $(C_d(\widetilde{A}))\alpha = [C_L, C_R]$.

$$d(\tilde{A}, C_d(\tilde{A})) = \sqrt{\int_0^1 (A_L(\alpha) - C_L)^2 d\alpha + \int_0^1 (A_R(\alpha) - C_R)^2 d\alpha}$$

with respect to C_L and C_R.

In order to minimize $d(\tilde{A}, C_d(\tilde{A}))$, it is sufficient to minimize the function

D (C_L, C_R)(= $d^2(\tilde{A}, C_d(\tilde{A}))$). The first partial derivatives are

$$\frac{\partial D(C_L, C_R)}{\partial C_L} = -2\int_0^1 A_L(\alpha) d\alpha + 2C_L$$

and $$\frac{\partial D(C_L, C_R)}{\partial C_R} = -2\int_0^1 A_R(\alpha) d\alpha + 2C_R$$

Solving $\frac{\partial D(C_L, C_R)}{\partial C_L} = 0 \quad and \quad \frac{\partial D(C_L, C_R)}{\partial C_R} = 0$ we get

$C_L^* = \int_0^1 A_L(\alpha)\ d\alpha$ and $C_R^* = \int_0^1 A_R(\alpha)\ d\alpha$

Again since

$$\frac{\partial D^2(C_L, C_R)}{\partial C_L^2} = 2 > 0, \quad \frac{\partial D^2(C_L, C_R)}{\partial C_R^2} = 2 > 0$$

$$H(C_L^*, C_R^*) = \frac{\partial D^2(C_L^*, C_R^*)}{\partial C_L^2} \cdot \frac{\partial D^2(C_L^*, C_R^*)}{\partial C_R^2} - \left(\frac{\partial D^2(C_L^*, C_R^*)}{\partial C_L\ \partial C_R} \right)^2 = 4 > 0$$

So D(C_L,C_R), i.e. $d(\tilde{A}, C_d(\tilde{A}))$, is global minimum. Therefore, the interval

$$C_d(\tilde{A}) = \left[\int_0^1 A_L(\alpha) d\alpha, \int_0^1 A_R(\alpha) d\alpha \right]$$ is the nearest interval approximation of fuzzy number $\tilde{A}$ with respect to the metric d.

Let $\tilde{A}$ = (a, b, c, d, e) be a Pentagonal fuzzy number. The α – level interval of $\tilde{A}$ is defined as $A_\alpha = [A_L(\alpha), A_R(\alpha)]$.

When $\tilde{A}$ is a linear fuzzy number (LFN), the left and right α cuts are

$$A_L(\alpha) = \begin{cases} a + \dfrac{\alpha(b-a)}{w_A} & if\ a \le x \le b \\ \dfrac{(\alpha - w_A)\ (c-b)}{(1-w_A)} + b & if\ b \le x \le c \end{cases}$$

$$A_R(\alpha) = \begin{cases} d - \dfrac{(\alpha - w_A)\ (d-c)}{(1-w_A)} & if\ c \le x \le d \\ e - \dfrac{\alpha(e-d)}{w_A} & if\ d \le x \le e \end{cases}$$

By the nearest interval approximation method, the lower and upper limits of the interval [C_L,C_R] are

$$C_L(\alpha) = \frac{1}{2(1-w_A)}\left[(b+c) + w_A(a-b-2c) - {w_A}^2(a-c)\right]$$

$$C_R(\alpha) = \frac{1}{2(1-w_A)}\left[(d+c) + w_A(e-d-2c) - {w_A}^2(e-c)\right]$$

5. THE PROPOSED INVENTORY MODEL IN FUZZY ENVIRONMENT:

If the cost parameters and total display shelf space parameters are fuzzy numbers, then the problem (1) to (3) is transformed to

$$Max\ \ \tilde{PF}(S,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} d_i\, S_i^{d_i'}(\tilde{p}_i - \tilde{C}_i) - \dfrac{\tilde{C}_{3i}}{Q_i} d_i\, S_i^{d_i'} \\ - \dfrac{(\tilde{g}_i + \tilde{C}_{1i})\, Q_i}{2} + \dfrac{d_i\, S_i^{d_i'}(\tilde{C}_{1i} + \tilde{g}_i) Q_i}{2\tilde{P}_i} \\ - (\tilde{g}_i + \tilde{C}_{1i} + \tilde{C}_{2i})\, S_i + \dfrac{d_i S_i^{d_i'+1}(\tilde{C}_{1i} + \tilde{g}_i)}{\tilde{P}_i} \end{array} \right] \tag{4}$$

$$Min\ \ \tilde{T}C(S,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} \dfrac{\tilde{C}_{3i}}{Q_i} d_i\, S_i^{d_i'} + \dfrac{(\tilde{g}_{si} + \tilde{C}_{s1i})\, Q_i}{2} \\ - \dfrac{d_i\, S_i^{d_i'}(\tilde{C}_{s1i} + \tilde{g}_{si}) Q_i}{2\tilde{P}_i} \\ + (\tilde{g}_{si} + \tilde{C}_{s1i} + \tilde{C}_{2i})\, S_i \\ - \dfrac{d_i S_i^{d_i'+1}(\tilde{C}_{s1i} + \tilde{g}_{si})}{\tilde{P}_i} \end{array} \right] \tag{5}$$

$$Min\ \ \tilde{W}C(S,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} \dfrac{(\tilde{g}_{wi} + \tilde{C}_{w1i})\, Q_i}{2} - \\ \dfrac{d_i\, S_i^{d_i'}(\tilde{C}_{w1i} + \tilde{g}_{wi}) Q_i}{2\tilde{P}_i} \\ + \dfrac{\tilde{t}_{wi} d_i\, S_i^{d_i'}}{Q_i} \end{array} \right] \tag{6}$$

where ~ represents the fuzzification of the parameters.
In our proposed model, the cost parameters are considered as pentagonal fuzzy number.

$\widetilde{P}_i = (P_{1i}, P_{2i}, P_{3i}, P_{4i}, P_{5i})$ $\widetilde{p}_i = (p_{1i}, p_{2i}, p_{3i}, p_{4i}, p_{5i})$,

$\widetilde{C}_i = (C_{1i}, C_{2i}, C_{3i}, C_{4i}, C_{5i})$, $\widetilde{C}_{2i} = (C_{21i}, C_{22i}, C_{23i}, C_{24i}, C_{25i})$

, $\widetilde{C}_{3i} = (C_{31i}, C_{32i}, C_{33i}, C_{34i}, C_{35i})$,

$\widetilde{C}_{w1i} = (C_{w11i}, C_{w12i}, C_{w13i}, C_{w14i}, C_{w15i})$,

$\widetilde{g}_{wi} = (g_{w1i}, g_{w2i}, g_{w3i}, g_{w4i}, g_{w5i})$,

$\widetilde{C}_{s1i} = (C_{s11i}, C_{s12i}, C_{s13i}, C_{s14i}, C_{s15i})$,

$\widetilde{g}_{si} = (g_{s1i}, g_{s2i}, g_{s3i}, g_{s4i}, g_{s5i})$

Using the Nearest Interval Approximation, the above model is defuzzified as follows

$$Max \;\; PF_C(S,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} d_i S_i^{d_i'} (p_{iC} - C_{iC}) \\ - \dfrac{C_{3iC}}{Q_i} d_i S_i^{d_i'} - \dfrac{(g_{iC} + C_{1iC}) Q_i}{2} \\ - (g_{iC} + C_{1iC} + C_{2iC}) S_i + \\ \dfrac{d_i S_i^{d_i'} (C_{1iC} + g_{iC}) Q_i}{2 P_{iC}} + \\ \dfrac{S_i^{d_i'+1} (C_{1iC} + g_{iC}) d_i}{P_{iC}} \end{array} \right] \quad (7)$$

$$Min \;\; TC_C(S,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} \dfrac{C_{3ic}}{Q_i} d_i S_i^{d_i'} + \\ \dfrac{(g_{sic} + C_{s1ic}) Q_i}{2} - \\ \dfrac{d_i S_i^{d_i'} (C_{s1ic} + g_{sic}) Q_i}{2 P_{ic}} \\ + (g_{sic} + C_{s1ic} + C_{2ic}) S_i \\ - \dfrac{d_i S_i^{d_i'+1} (C_{s1ic} + g_{sic})}{P_{ic}} \end{array} \right] \quad (8)$$

$$Min \;\; WC_C(S,Q) = \sum_{i=1}^{n} \left[\begin{array}{l} \dfrac{(g_{wic} + C_{w1ic}) Q_i}{2} \\ - \dfrac{d_i S_i^{d_i'} (C_{w1ic} + g_{wic}) Q_i}{2 P_{ic}} \\ + \dfrac{t_{wic} d_i S_i^{d_i'}}{Q_i} \end{array} \right] \quad (9)$$

Step-1: Solve the multi- objective programming problem considered as a single objective problem, which is taking only one objective function at a time and ignoring the rest objective function. By using geometric programming technique the optimal solution is derived. Let X^i be the optimal solution for the i^{th} single objective problem.

Step-2: From the results of step-I, determine the corresponding values for every objective function at each optimal solution derive. Using all the above optimal values of the objective function in step-1, construct a pay-off matrix (3 x 3) as follows:

$$\begin{array}{cccc} & PF & TC & WC \\ X^1 & PF_C^1 & TC_C^1 & WC_C^1 \\ X^2 & PF_C^2 & TC_C^2 & WC_C^2 \\ X^3 & PF_C^3 & TC_C^3 & WC_C^3 \end{array}$$

From the payoff matrix, lower bound of objective functions are

$$L_{PF_C} = Min[PF_C^1, PF_C^2, PF_C^3], L_{TC_C} = Min[TC_C^1, TC_C^2, TC_C^3],$$
$$L_{WC_C} = Min[WC_C^1, WC_C^2, WC_C^3]$$

and the upper bound of objective functions are

$$U_{PF_C} = Max[PF_C^1, PF_C^2, PF_C^3], U_{TC_C} = Max[TC_C^1, TC_C^2, TC_C^3],$$
$$W_{WC_C} = Max[WC_C^1, WC_C^2, WC_C^3]$$

$$L_{PF_C} \le PF_C \le U_{PF_C}, L_{TC_C} \le TC_C \le U_{TC_C},$$
$$L_{WC_C} \le WC_C \le U_{WC_C},$$

Step- 3:
From step- 2, the membership functions $\mu_{PF_C}(S,Q)$, $\mu_{TC_C}(S,Q)$ and $\mu_{WC_C}(S,Q)$ are constructed below

$$\mu_{PF_C}(S,Q) = \begin{cases} 1 & PF_C(S,Q) \ge U_{PF_C} \\ 1 - \dfrac{U_{PF_C} - PF_C(S,Q)}{U_{PF_C} - L_{PF_C}} & L_{PF_C} \le PF_C(S,Q) \le U_{PF_C} \\ 0 & Otherwise \end{cases}$$

$$\mu_{TC_C}(S,Q) = \begin{cases} 1 & TC_C(S,Q) \ge U_{TC_C} \\ 1 - \dfrac{U_{TC_C} - TC_C(S,Q)}{U_{TC_C} - L_{TC_C}} & L_{TC_C} \le TC_C(S,Q) \le U_{TC_C} \\ 0 & Otherwise \end{cases}$$

$$\mu_{WC_C}(S,Q) = \begin{cases} 1 & WC_C(S,Q) \ge U_{WC_C} \\ 1 - \dfrac{U_{WC_C} - WC_C(S,Q)}{U_{WC_C} - L_{WC_C}} & L_{WC_C} \le WC_C(S,Q) \le U_{WC_C} \\ 0 & Otherwise \end{cases}$$

The problem (7) to (9) can be formulated as

Max V(S, Q) = $[\mu_{PF_C}(S,Q) + \mu_{TC_C}(S,Q) + \mu_{WC_C}(S,Q)]$

omitting constant terms,

ie) $\text{Max } V(S,Q) = \sum_{i=1}^{n}\left[k_{1i}S_i^{d_i^1} - k_{2i}\frac{S_i^{d_i^1}}{Q_i} - k_{3i}Q_i + k_{4i}S_i^{d_i^1}Q_i - k_{5i}S_i + k_{6i}S_i^{d_i^1+1} - \frac{k_{7i}}{Q_i}\right]$

The standard geometric programming problem is

$$\text{Min } V(S,Q) = \sum_{i=1}^{n}\left[-k_{1i}S_i^{d_i^1} + k_{2i}\frac{S_i^{d_i^1}}{Q_i} + k_{3i}Q_i - k_{4i}S_i^{d_i^1}Q_i + k_{5i}S_i - k_{6i}S_i^{d_i^1+1}\right] \tag{10}$$

This primal problem (10) is a signomial problem with 4n-1 degree of difficulty. The corresponding dual problem is

$$Max \quad V_L = \left[\prod_{i=1}^{n}\left(\frac{k_{1i}}{w_{1i}}\right)^{-w_{1i}}\left(\frac{k_{2i}}{w_{2i}}\right)^{w_{2i}}\left(\frac{k_{3i}}{w_{3i}}\right)^{w_{3i}}\left(\frac{k_{4i}}{w_{4i}}\right)^{-w_{4i}}\left(\frac{k_{5i}}{w_{5i}}\right)^{w_{5i}}\left(\frac{k_{6i}}{w_{6i}}\right)^{-w_{6i}}\right]$$

Subject to:

$-w_{1i} + w_{2i} + w_{3i} - w_{4i} + w_{5i} - w_{6i} = 1$

$-d_i' w_{1i} + d_i' w_{2i} - d_i' w_{4i} + w_{5i} - (d_i' + 1)w_{6i} = 0$

$-w_{2i} + w_{3i} - w_{4i} = 0$

where $w_{1i}, w_{2i}, w_{3i}, w_{4i}, w_{5i}$ & $w_{6i} > 0$.

By using geometric programming theorem , the analytical expressions for the decision variables Q_i and S_i are obtained

$$Q_i^* = \frac{w_{4i}k_{1i}}{w_{1i}k_{4i}} \tag{11}$$

$$S_i^* = \frac{k_{3i}Q_i^* w_{5i}}{w_{3i}k_{5i}} \tag{12}$$

6. NUMERICAL EXAMPLE

Assuming an Apparel showroom they sell 2 items. The relevant data for the two items is given below:

$D_1 = 20S_1^{0.5}$ units, C_1 =₹500, C_{w11}=₹110, C_{s11}=₹80, C_{21} =₹140, C_{31} =₹500, p_1 =₹700, g_{w1}=₹1,

g_{s1}=₹5, P_1=2400 units, t_{w1}=₹200, $D_2 = 500S_2^{0.1}$ units, C_2 =₹100, C_{w12}=₹ 60, C_{s12}=₹ 80, C_{22} =₹140,

C_{32} =₹ 200, p_2 =₹ 1000, g_{w2}=₹5, g_{s2}=₹9, P_2=3000 units, t_{w2}=₹100

$\widetilde{C_1} = [500\ 600\ 700\ 800\ 900]$,

$\widetilde{C_{w11}} = [110\ 120\ 130\ 140\ 150]$

$\widetilde{C_{s11}} = [80\ 85\ 90\ 95\ 100]$

$\widetilde{C_{21}} = [140\ 141\ 142\ 143\ 144]$

$\widetilde{C_{31}} = [500\ 510\ 520\ 530\ 540]$,

$\widetilde{p_1} = [700\ 1000\ 1300\ 1600\ 1900]$

$\widetilde{g_{w1}} = [1\ 2\ 3\ 4\ 5]$

$\widetilde{g_{s1}} = [5\ 5.5\ 6\ 6.5\ 7]$

$\widetilde{P_1} = [2400\ 2500\ 2600\ 2700\ 2800]$

$\widetilde{C_2} = [100\ 105\ 110\ 115\ 120]$

$\widetilde{C_{w12}} = [60\ 65\ 70\ 75\ 80]$

$\widetilde{C_{s12}} = [80\ 85\ 90\ 95\ 100]$

$\widetilde{C_{22}} = [140\ 141\ 142\ 143\ 144]$

$\widetilde{C_{32}} = [200\ 210\ 220\ 230\ 240]$

$\widetilde{P_2} = [3000\ 3500\ 4000\ 4500\ 5000]$,

$\widetilde{p_2} = [1000\ 1100\ 1200\ 1300\ 1400]$,

$\widetilde{g_{w2}} = [5\ 6\ 7\ 8\ 9]$

$\widetilde{g_{s2}} = [9\ 9.1\ 9.2\ 9.3\ 9.4]$

$\widetilde{t_{w1}} = [200\ 205\ 210\ 215\ 220]$,

$\widetilde{t_{w2}} = [100\ 110\ 120\ 130\ 140]$, $w_A = 0.75$

Table 1: Optimal Solutions

Cases	i	Si*(units)	Qi*(units)	PF(S*,Q*)(Rs.)	TC(S*,Q*)(Rs.)	WC(S*,Q*)(Rs.)
Crisp	1	87	172	8,031	90,192	20,016
	2	233	467			
Fuzzy	1	66	105	19,596	97,054	15,147
	2	274	342			

7. OBSERVATION

From table 1, it should be noted that Compared to crisp model the fuzzy model is very effective method. The average Profit is obtained in fuzzy model is higher than the crisp model as well as the average total cost for store and warehouse is less than the crisp model.

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